Nonlinear control synthesis of biomechanical sit to stand movement

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Abstract— **Sit-to-stand (STS) movement is a complex functional task which requires coordination of movement, postural regulation and stability for successful execution. STS transfer consolidates the highly nonlinear musculoskeletal structure together with neural control and tactile system in human body. In this paper we propose a nonlinear control technique based on feedback linearization to emulate the control action of central nervous system in performing STS movement. We use 4-segments rigid body biomechanical model with 3 degrees-of-freedom built on average anatomical proportions. In this controlling scheme, we adopt the output feedback computing through physiologically relevant optimization based upon center of mass (COM) and ground reaction forces (GRF). Furthermore, the output feedback provides passive control action commands including a linear quadratic regulator (LQR) based function augmented with nonlinear function computed with feedback linearization. The reference trajectories generate active feedforward torques, in addition with passive torques to settle the motion profiles within human anatomical constraints. The simulation results show that feedback linearization in combination with LQR provides an optimal frame work for better results of biomechanical STS movement as compared to previous linear control design schemes.**

Keywords— Feedback linearization; nonlinear control; biomechanical model; Sit to stand movement.

I. INTRODUCTION

Performing sit to stand task is a standout amongst the most well-known assignments in day to day life. The STS task can be considered as the prerequisite for performing various activities. Sit to stand capacity is a key factor and marker in practical autonomy. Execution of sit to stand brings about physiological changes from a stable sitting position to a less stable standing position with a higher COM and small base of support (BOS) because of augmentation of the lower extremities. Thus, to perform STS transfer successfully higher joint torques and in addition exact control of COM movement inside the base of support is required. STS movement execution requires coordination among skeletal structure, muscles, ligaments or tendons. The central nervous system (CNS) issues command signals as per feedback from muscle tendon actuators and vestibular sensors that give data about body orientation. The researchers have studied this movement in different perspectives for different understanding using biomechanical models.

Clinical analysts in [1] discussed the STS movement from physiological perspective and categorized 17 determinants of movement. These determinants were ordered into four subjects related (e.g. muscles, age), four seat-related (e.g. chair height, arms support), and eleven strategies related (e.g. speed of movement, foot position). They noticed the need of control factors which impact the STS movement execution. Their discussion included the experimental arrangements including force plates, optoelectronics strategies, movement video capturing schemes, and accelerometers. Physiologists mostly describe STS movement in distinctive phases. Experimental data shows that STS movement contains two principle stages: the forward thrust stage and the upward extension stage [2]. Researchers in [3] ordered and examined the sit to stand movement into additional stages. In the forward extension stage lifting off from seat comprises of forward flexion of head-arm-trunk (HAT) and forward and upwards movement of shank. The upward extension stage comprises of maximum flexion and dorsiflexion of hip and lower leg joint. Augmentation of hip and knee joints and plantar flexion of the lower leg joint balances out the body in an upright position. Researchers in [4] gathered kinematic date for STS transfer from 50 healthy subjects ages (2~ 78) including both male and females. The mean time for complete motion was calculated to be 1.907 ±0.057 sec. Researchers in [5-6] reported that COM, center-of-pressure (COP), head-position (HP) and GRF aimed the STS transfer place constraints in movement coordination of joints. The authors in ref [7] confirmed that setting the COM over the foot and tailing it with upward movement makes it simpler for the people to accomplish this task through minimal coordination of joints. Researchers in ref [8] formulated the assisted and unassisted STS movement as an optimal control problem. They employed the differential dynamic programming to determine the optimal STS movement control strategy. The authors successfully implemented the strategy on assistive robots. The researchers in [9] quantified the STS movement using the GRF and COP estimated from Nintendo Wii Balance Board. The information was gathered from 503 subjects (male/female, 226/277), ages (20~80). The STS score was figured as the combination of speed and balance indices. They reported that the proposed STS score will be valuable to identify the early disintegration of motor performance. Iqbal and Mughal in ref [10] studied the active and passive mechanism for postural control and STS movement using 4-segments biomechanical model. The authors optimized physiological costs with LQR when following predefined trajectories for ankle, knee, and hip joints. The similar model was studied for balance recovery, fall prevention and STS movement analysis [11]. However, the linear controllers were used to emulate the action of CNS. This model was further studied for STS motion analysis using TSK fuzzy model combined with H_2 and H_∞ control techniques [12-14]. The model was linearized at sitting and standing position. The modeling scheme then interpolates between two linearized models, using the triangular and Gaussian membership functions for optimal control design. These modeling schemes required more time 4-10 sec for complete STS movement with large deviation in angular profiles. The 4-segments biomechanical model with inertial and gravitational components is a nonlinear model. For optimal control the

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linearization of model is required which leads to linearization error in model due to unmolded dynamics. The functional region of linear controller is limited to small neighborhood of equilibrium points. These problems can be taken care of, with using nonlinear control technique. Our current work is the extension of previous work done to overcome the problem of linearization error as well as to design a controller that is valid for all functional regions. In this paper we propose a better controlling scheme based on feedback linearization in augmentation with LQR as compared to simple linear control techniques presented in literature. We simulate the biomechanical model comprising three joints (ankle, knee and hip) and four segments for STS transfer. The cost functions are based upon physiological variables COM and GRF. Feedback linearization is a power full method that enables us to acquire linear system with exact dynamics. The transformed linear system is then valid for all functional regions [15]. This technique relies on picking new state variables and state inputs. The nonlinear feedback inputs then compensate for nonlinearities in state equations. The active and passive components of joint torque are generated by reference trajectories and the feedback of states respectively. Our results intend to track the reference trajectories with minimum torques and without undesired deviation in angular profiles as compared to previous work done with simple LQR [10,16] and Fuzzy combined with LQR [17].

II. FORMULATION OF BIOMECHANICAL MODEL

A. Four-Segments Sagittal plane Biomechanical Model

In our research we model the human body as a multi segment structure containing anatomical skeletal, muscular and sensory subsystems. The four segments shown in Fig.1 represent bilateral symmetrical arrangement of feet (BOS), shank (lower limb), thighs (upper limb) and HAT in the sagittal plan. All segments are connected by single degree of freedom (DOF) which represents a total of 6 DOF in rigid body mechanics. F_x and F_y represent horizontal and vertical ground reaction forces on base of support. Other physical parameters are mass m_i, length l_i and inertia I_i of each segment. The joint angles θ_i represent the ankle, knee and hip posture positions and are measured from horizontal x-axis. τ_i represents the toque applied at each joint. The human voluntary movements involve both active and passive mechanism [18]. CNS commands form higher centers or reflex loop generate the muscular forces which then convert into torque actuation at ankle, knee and hip joints. The torque input at ankle, knee and hip joint is the sum of active and passive torques. Active or feed-forward torques are generated in response to CNS commands and are task specific. Passive or feedback torques from CNS are generated because of intrinsic stiffness or viscosity in muscle tendon structures.

B. Dynamic equations of Model

The four segments biomechanical model shown in Fig.1 is a nonlinear model represented by the following equation.

$$D(\theta)\ddot{\theta} + H(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \vec{\tau} \tag{1}$$

Where θ represents the joint angle vector, $D(\theta)$ is the inertial component matrix of joint moments, $H(\theta)$ and $G(\theta)$ represents the coriolis and gravitational component matrices respectively

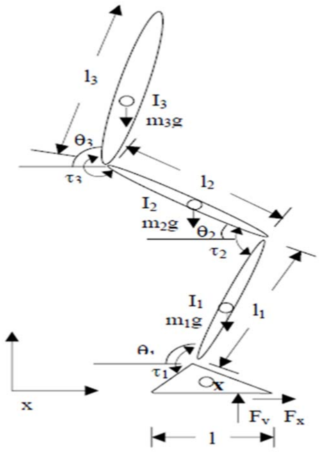

Fig.1 Four segments Biomechanical model

given in (Apendix.1). The torque vector in eq.1 is $\vec{\tau} = [\tau_1 - \tau_2 \quad \tau_2 - \tau_3 \quad \tau_3]^T$ which represents input torques at ankle knee and hip joints respectively. Whereas the total torque at each joint is the sum of feedforward τ_{ff} and feedback τ_{fb}. τ_{fb} generated by feedback of states, ensures that error with respect to assumed reference trajectories will go to zero with passage of time. The Non-linear ODE defined in equation (1) can be written as

$$\ddot{\theta} = D(\theta)^{-1}\left[\vec{\tau} - H(\theta,\dot{\theta})\dot{\theta} - G(\theta)\right] \tag{2}$$

We can now define the state variables as

$$x = [\theta_1\ \theta_2\ \ \theta_3\ \theta_4\ \theta_5\ \theta_6\ ]^T$$

The non-Linear state space equation is given by

$$\dot{x}_1 = x_4\ ,\ \dot{x}_2 = x_5\ ,\ \dot{x}_3 = x_6$$

$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{bmatrix} = \begin{bmatrix}\dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = [D^{-1}]\left[-H\begin{bmatrix}x_4\\ x_5\\ x_6\end{bmatrix} - G + \tau\right] \tag{3}$$

We defined x_1, x_2, x_3 as position states and $x_4\ x_5\ x_6$ as velocity states. Thus, nonlinear system is a function of states and inputs represented as $\gamma(x_1\ x_2\ x_3\ x_4\ x_5\ x_6\ \tau_1\ \tau_2\ \tau_3)$. The nonlinear state space formulation defined in eq.3 is of the form

$$\dot{x} = A(x) + j_1(x)u_1 + j_2(x)u_2 + j_3(x)u_3\ ,\quad y = h(x) \tag{4}$$

Where $A(x)$ defines nonlinear system equations and j_1, j_2, j_3 are nonlinear input functions represented as follows:

$$A = \begin{bmatrix} x_4\\ x_5\\ x_6\\ \gamma_1(x_1,x_2,x_3,x_4,x_5,x_6)\\ \gamma_2(x_1,x_2,x_3,x_4,x_5,x_6)\\ \gamma_3(x_1,x_2,x_3,x_4,x_5,x_6)\end{bmatrix},\ j = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ j_{41} & j_{42} & j_{43}\\ j_{51} & j_{52} & j_{53}\\ j_{61} & j_{62} & j_{63}\end{bmatrix}$$

Where u_1, u_2, u_3 represent input torques at ankle, knee and hip joints, and are nonlinear functions. For STS transfer the ankle, knee and hip posture positions are of interest so we define output as: $y = [x_1 \quad x_2 \quad x_3]^T$

The nonlinear model defined in eq.4 is an open loop unstable system. State feedback control input is required for stable body movement to accomplish STS task.

III. Controller Design

A. Feedback Linearization

In this paper we use tracking control via feedback linearization combined with LQR to construct control for biomechanical model to perform STS transfer. This technique comprises of two parts. One part containing the nonlinear feedback gains cancel the nonlinearities in dynamic equations and the other part controls the subsequent linear system. We assume that $A(0) = 0$ and $j(0) = 0$. Where functions $A(x), j(x)$ and $h(x)$ are smooth and sufficiently many times differentiable in the domain D. Then the mapping A and j are called vector fields on D. The system has vector relative degree $[r_1 \quad r_2 \quad r_3] = [2 \quad 2 \quad 2]$. With this relative degree the system has following dynamic equations. Where L represents lie derivatives.

$$\dot{x}_1 = x_4 \,, \; \dot{x}_2 = x_5 \,, \dot{x}_3 = x_6$$

$$\dot{x}_4 = L_\gamma^2 h_1(x) + \left(L_{j_1} L_\gamma h_1(x)\right) u_1(x) + \left(L_{j_2} L_\gamma h_1(x)\right) u_2(x)$$

$$+ \left(L_{j_3} L_\gamma h_1(x)\right) u_3(x)$$

$$\dot{x}_5 = L_\gamma^2 h_2(x) + \left(L_{j_1} L_\gamma h_2(x)\right) u_1(x) + \left(L_{j_2} L_\gamma h_2(x)\right) u_2(x)$$

$$+ (L_{j_3} L_\gamma h_2(x)) u_3(x)$$

$$\dot{x}_6 = L_\gamma^2 h_3(x) + \left(L_{j_1} L_\gamma h_3(x)\right) u_1(x) + \left(L_{j_2} L_\gamma h_3(x)\right) u_2(x)$$

$$+ \left(L_{j_3} L_\gamma h_3(x)\right) u_3(x) \qquad (5)$$

For convenience of notation let

$$\beta_1(x) = L_\gamma^2 h_1(x), \beta_2(x) = L_\gamma^2 h_2(x), \beta_3(x) = L_\gamma^2 h_3(x)$$

$$\alpha_{11}(x) = \left(L_{j_1} L_\gamma h_1(x)\right), \alpha_{12}(x) = \left(L_{j_2} L_\gamma h_1(x)\right)$$

$$\alpha_{13}(x) = \left(L_{j_3} L_\gamma h_1(x)\right), \alpha_{21}(x) = \left(L_{j_1} L_\gamma h_2(x)\right)$$

$$\alpha_{22}(x) = \left(L_{j_2} L_\gamma h_2(x)\right), \alpha_{23}(x) = \left(L_{j_3} L_\gamma h_2(x)\right)$$

$$\alpha_{31}(x) = \left(L_{j_1} L_\gamma h_3(x)\right), \alpha_{32}(x) = \left(L_{j_2} L_\gamma h_3(x)\right)$$

$$\alpha_{33}(x) = \left(L_{j_3} L_\gamma h_3(x)\right)$$

Putting these notations in eq.5 we represent the equations in simplified form as:

$$u = \begin{bmatrix} u_1(x) \\ u_2(x) \\ u_3(x) \end{bmatrix}, B = \begin{bmatrix} \beta_1(x) \\ \beta_2(x) \\ \beta_3(x) \end{bmatrix}, A = \begin{bmatrix} \alpha_{11}(x) & \alpha_{12}(x) & \alpha_{13}(x) \\ \alpha_{21}(x) & \alpha_{22}(x) & \alpha_{23}(x) \\ \alpha_{31}(x) & \alpha_{32}(x) & \alpha_{33}(x) \end{bmatrix}$$

We calculate the control input torque using following equation.

$$u = A^{-1}(x)[-B(x) + v] \qquad (6)$$

In above equation $v = -kx_i \in R^{6\times 6}$. Where k represents the linear optimal feedback gains. The control input torque represented by u in eq.6 is the passive torque generated at respective joints. The nonlinear gains cancel the nonlinearities in dynamic equation by multiplication and addition [19]. In all previous studies [7,10-13,16] the nonlinear terms were ignored by linearizing the model at standing position leading to larger deviations in angular profiles. In TSK fuzzy modeling [12,14] two controller gain matrices were used by linearizing the model at sitting and standing positions. This modeling scheme is thus computationally expansive. For optimal control we need to choose k such as to stabilize the system. We use LQR to calculate linear gains to optimize the system performance. LQR is an optimal controller that operates the dynamic system at minimum cost. Riccati equation for this design problem is given as:

$$-\dot{M} = MA + A^T M - MBR^{-1}B^T M + Q \qquad (7)$$

The choice of weighting matrices Q and R are the key elements in controller performance. These are normally selected as diagonal matrices, where all the diagonal elements are selected arbitrarily using trial and error for best performance. However, in case of biomechanical model it is desirable that the selection of control weights must relate physiological phenomena.

B. Physiological Cost functions

Two physiological variable COM and GRF are considered for selection of optimal control weights. Both physiological variables put constraints on movement coordination of STS transfer. The COM equations are given as [11]:

$$x_{com} = l_f - a + \frac{\sum_i^3 f_i \cos x_i - cmf}{m+mf}, \; y_{com} = b + \frac{\sum_i^3 f_i \sin x_i - \frac{b}{2}mf}{m+mf}$$

$$V_{xcom} = \frac{\sum_i^3 f_i(\sin x_i) x_{i+3}}{m+mf}, \qquad V_{ycom} = \frac{\sum_i^3 f_i(\cos x_i) x_{i+3}}{m+mf} \qquad (8)$$

All the parameters in eq.8 are defined in table.1. The position and velocity sensitivity derivatives are calculated at standing position $\left[\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}, 0,0,0\right]^T$ corresponding to θ and $\dot{\theta}$ states. The diagonal terms of Q are calculated as

$$q_{ii} = \left|\frac{\partial x_{com}}{\partial x_i}\right|^2_{x=x_e, u=u_e} \quad (i = 1,2,3)$$

$$q_{ii} = \left|\frac{\partial v_{com}}{\partial x_i}\right|^2_{x=x_e, u=u_e} \quad (i = 4,5,6) \qquad (9)$$

The COM equations do not generate weights for inputs. The GRF equations are given by

$$F_x = \sum_{i=1}^{3} f_i(\ddot{x}_i \sin x_i + \dot{x}_i^2 cos x_i)$$

$$F_y = \sum_{i=1}^{3} f_i(\ddot{x}_i cos x_i - \dot{x}_i^2 sin x_i) + (m + mf)g \qquad (10)$$

In above equation x_i and $\dot{x}_i$ represent position and velocity state variables as defined earlier. Where $\ddot{x}_i$ represents the nonlinear equation defined in (3). We evaluate sensitivity derivatives at standing position and generate weights for angle state variables and joint input torques. The movement termination at standing posture results in zero weights for the velocity states. The GRF advantage is that it calculates weights for both states and inputs. The Q and R diagonal terms are calculated as

$$q_{ii} = \left|\frac{\partial F_x}{\partial x_i}\right|^2_{x=x_e, u=u_e} \quad (i = 1,2,3)$$

$$r_{ii} = \left|\frac{\partial F_x}{\partial \tau_i}\right|^2_{x=x_e, u=u_e} \quad (i = 1,2,3) \qquad (11)$$

The off-diagonal terms are taken zero for independent minimization. We use hybrid scheme for LQR design which involves combination of COM and GRF based optimization. The optimal state and input weights are selected as

$$Q_h = Q_{com} + Q_{GRF}, \quad R_h = R_{GRF} \qquad (12)$$

C. *Active or Feed forward Torques*

Feed forward torques are generated from reference trajectories. We evaluate the nonlinear model in eq.2 at equilibrium points x_e (standing position). These toques are equal to equilibrium torque u_e and are only due to gravitational components as defined in following equation.

$$\tau_{ff} = g.\begin{bmatrix} f_1\cos(x_1) + f_2\cos(x_2) + f_3\cos(x_3) \\ f_2\cos(x_2) + f_3\cos(x_3) \\ f_3\cos(x_3) \end{bmatrix}.x_r \qquad (13)$$

In above equation x_r defines reference trajectories. The feedforward torque component by eq.13 and feedback torque component by eq.6 collectively constitute the total torque input to the nonlinear plant.

D. *Reference trajectories*

The nonlinear plant in presence of controlled input torque is a closed loop stable system and it settles to zero reference. In order to mimic point to point movement, reference trajectory model is employed as proposed by ref [20]. These reference trajectories resemble the experimental data for STS transfer. The individual reference trajectories are defined in general form as:

$$\vartheta_{ref}(t) = x_0 + (x_f - x_0).x_R(t) \qquad (14)$$

In above equation x_0 and x_f are initial and final posture positions in terms of angles. Reference trajectories defined by x_R are calculated form output of an open loop stable system as:

$$\dot{x}_R(t) = A_R x_R(t). \text{ Where } \quad A_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\mu & -\rho \end{bmatrix} \qquad (15)$$

μ and ρ are scaled for faster or slower response. The knee joint trajectory starts from $x_2 = 0$ and ends at $x_2 = \frac{\pi}{2}$. It resembles a sigmoid function. The ankle and hip joint angles start at $\frac{\pi}{2}$ and then ends at $\frac{\pi}{2}$. To track the reference angles, we modified eq.6 as state feedback tracking control signal. For a system with relative degree 2 the tracking control signal with bounded derivative is given as:

$$u = A^{-1}(x)[-b(x) + v + \ddot{x}_R] \qquad (16)$$

The above equation achieves asymptotic stability with zero tracking error for reference angles.

IV. SIMULATION RESULTS

To ascertain the viability of our modelling scheme we simulate our model in Matlab/Simulink. Simulation results are based upon nonlinear dynamic model in eq.1-2 with nonlinear controller in (16). The simulation scheme for STS transfer is shown in Fig.2.

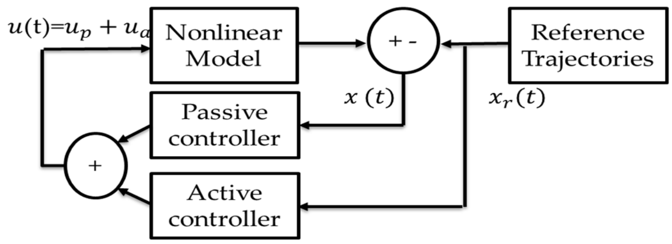

Fig.2 Simulation scheme for four segments biomechanical model

Sit to stand movement coordination requires determination of ankle, knee and hip joint trajectories. Reference trajectories are shown in Fig.3 and are generated using eq.14. These trajectories are plotted according to desired starting $[\frac{\pi}{2}, 0, \frac{\pi}{2}]$ and terminal $[\frac{\pi}{2}, \frac{\pi}{2}, \frac{\pi}{2}]$ position. The angular position error profile is shown in Fig.4. We assumed zero initial angular velocity at movement initiation of STS. This means that seat off has taken place earlier and we are not taking into consideration any seat or hands reaction forces. The ankle and hip joint profile starts and end at $\theta = \frac{\pi}{2}$. The HAT moves in forward direction during forward thrust phase and then upward extension phase commence. The knee joint profile starts at $\theta = 0$ and ends at $\theta = \frac{\pi}{2}$. Our simulation results show that joint profiles are in phase with reference trajectories. The hip joint trajectory is out of phase in case of previous study [10,16] with simple LQR and with fuzzy LQR design [17]. All profiles settle in approximately 1.7sec. Our results intend to track the reference angles in minimum time and with negligible deviation in angular profiles as compared to previous study [10-12,16,17]. Fig.5 shows the active and passive

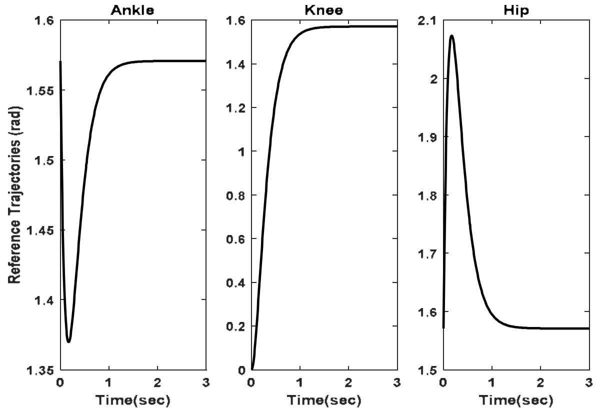

Fig.3 Reference Trajectory profiles for STS movement

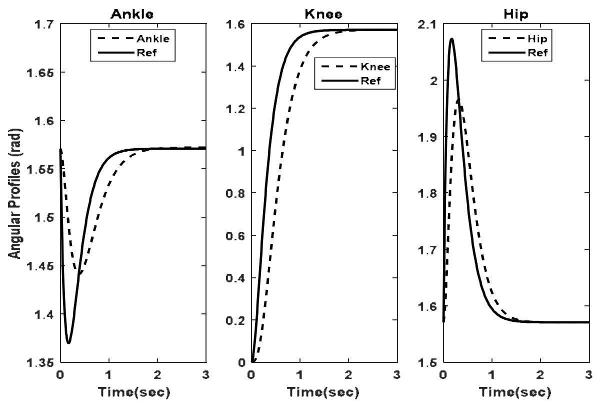

Fig.4 Joint Angular profiles for ankle, knee and hip joint

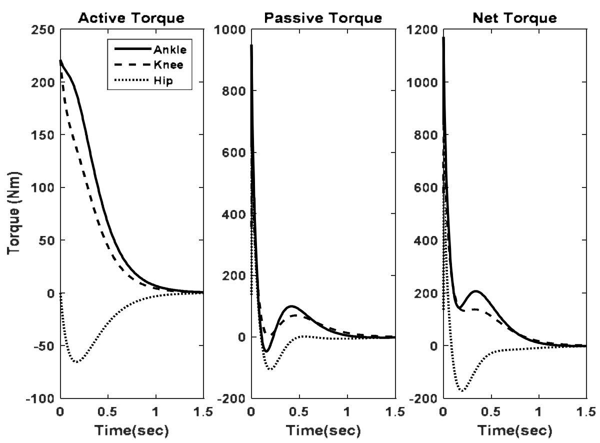

Fig.5 Active, Passive and Net Torque Profiles

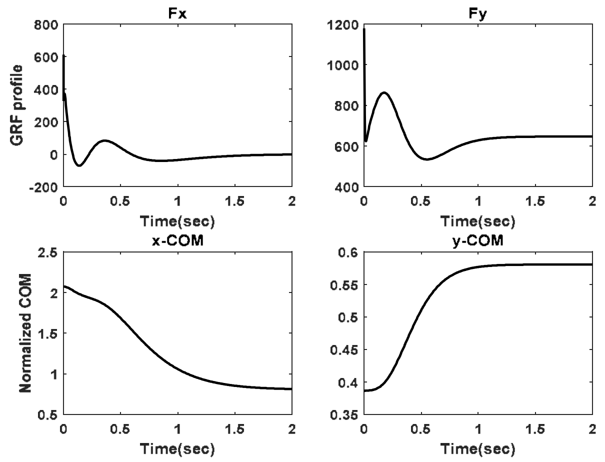

Fig.6 GRF (Fx, Fy) and COM (x-COM, y-COM) profile

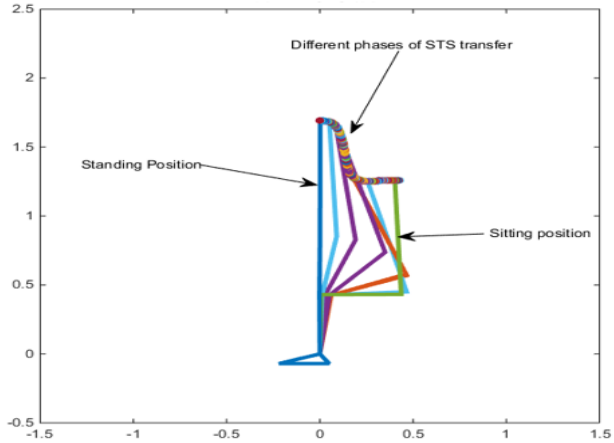

Fig.7 Profile of head trajectory and three link movements during STS task.

Maximum change in angle for healthy persons (rad)	Ref [10] LQR	Ref [16] LQR	Ref [17] Fuzzy with LQR	Ref Trajectories	Our simulations
Ankle	0.15	0.67	0.4	0.2	0.13
Knee	1.57	1.57	2	1.57	1.57
Hip	0.10(out of phase	0.31(out of phase	0.37(out of phase)	0.48	0.43
Settling time (s)	4	3.5	4	1.5	1.7

Fig.8 Comparison of our simulation results with Ref [10], [16], [17].

torque profiles generated with feedback linearization combined with hybrid LQR scheme. It is obvious from these profiles that the net hip joint torque stays negative during STS movement. This negative torque shows the extensor torque i.e. the flexion-torque followed by extension torque to facilitate the upward movement in extension phase. It is noticeable that passive torques are higher as compared to assumed active torque by CNS for STS movement. Fig.6 shows the horizontal and vertical-components of ground reaction forces. The GRF_x settles at zero, whereas the GRF_y settles at body weight 650N. Fig.6 also show the normalized COM_x and COM_y profiles. The COM_x profile is normalized at foot length l_f and it settles at 40% of foot length. COM_y profile is normalized at body height $(l_1 + l_2 + l_3 + b)$ and it settles at 57% of body height. In Fig.7 the head trajectory movement in coordination with 3-segments shows the physiologically correct STS transfer from seat off to stance position. Fig.8 shows the comparison of our results with previous work done. These results show that our simulation scheme provides an optimal framework for STS movement as compared to previous work done.

V. CONCLUSION

In our controlling scheme we emulate CNS as a nonlinear controller to carry out STS movement. For cost optimization we combined feedback linearization with physiological based LQR. We further studied the role of feedforward process (active control) and feedback process (passive stiffness) in performance of STS transfer. The STS movement analysis with feedback linearization provides new prospects to this research. This vector filed method combined with physiological LQR design provide better controlling approach as compared to linear control design schemes. Experimental results [5-6] show that STS movement coordination is reliant upon physiological variables. Thus, our controlling scheme is both cognitive in nature and optimal for measure of effort to achieve its undertaking objective. This study is useful for rehabilitation robotics. The STS movement disorders linked with kinematic senses can be diagnosed, quantified and rehabilitated in a better way. In future, applications for example prosthetics, implants, and exoskeleton designs will utilize frameworks that are cognitive controlled as well as optimal in nature to establish natural voluntary movements. Accordingly, we will extend our study to design a robust nonlinear controller for better control of coordinated movements. We will further extend our study by introducing neural delays at all joint angles and joint torques.

APPENDIX

Table.1 Parametric values of Biomechanical elements

No	Physical Parameters		
1	Foot mass (kg)	m_f	1.91
2	Foot length (m)	l_f	0.27
3	Ankle heel (m)	a	0.05
4	Ankle height (m)	b	0.07
5	Ankle-foor COM (m)	c	0.08
6	Gravity $g(m/s^2)$	g	9.8
7	Mass of model (kg)	(m)	64.09

Table.2 Physical parameters of biomechanical model

$$G = \left[g[f_1\cos(\theta_1)\ f_2\cos(\theta_2)\ \ f_3\cos(\theta_3)\right]^T$$

$$d_{11} = m_1k_1^2 + (m_2+m_3)l_1^2 + I_1\ ,\ d_{33} = m_3k_3^2 + I_3\ ,\ d_{12} = f_2l_1$$

$$d_{13} = f_3l_1\ ,\ d_{23} = f_3l_2$$

$$f_1 = m_1k_1 + (m_2+m_3)l_1\ ,\ f_2 = m_2k_2 + m_3l_2\ ,\ f_3 = m_3k_3$$

$$D = \begin{bmatrix} d_{11} & d_{12}\cos(\theta_1-\theta_2) & d_{13}\cos(\theta_1-\theta_3 \\ d_{12}\cos(\theta_1-\theta_2) & d_{22} & d_{23}\cos(\theta_2-\theta_3) \\ d_{13}\cos(\theta_1-\theta_3) & d_{23}\cos(\theta_2-\theta_3) & d_{33} \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & d_{12}\theta_5\sin(\theta_1-\theta_2) & d_{13}\theta_6\, sin(\theta_1-\theta_3) \\ -d_{12}\theta_4\sin(\theta_1-\theta_2) & 0 & d_{23}\theta_6\sin(\theta_2-\theta_3) \\ -d_{13}\theta_4\sin(\theta_1-\theta_3) & -d_{23}\theta_6\sin(\theta_2-\theta_3) & 0 \end{bmatrix}$$

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